QUANTIZATION OF THE ENERGY DENSITY IN A CLOSED UNIVERSE

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The energy of a closed universe is represented as a difference of two positive definite quantities, one of which includes the energy of matter and the energy of gravitational waves on the expanding universe background. The second quantity relates to the universe expansion and is called the energy of space. The whole energy of the universe equals zero provided the classical gravitational constraints are taken into account. In quantum theory a principle of the energy density of the matter minimum is formulated in the condition that the quantum gravitational constraints are also fulfilled in average. The states of the universe which satisfy the conditional minimal principle have different degree of the physical degrees of freedom excitation and, according to the gravitational constraints, corresponding excitation of space. The state of minimal excitation is proposed to be taken as the Beginning of the universe, and all the set of solutions, correspondingly, as admitted physical states of the universe at different moments of a cosmic time.

Keywords: energy; time; expanding universe; gravitational constant; quantum state; reference frame

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Introduction

The Wheeler – de Witt equation (WdW) [1, 2] is the cornerstone of modern quantum cosmology. The equation can be written in the following condensed form:

$$\widehat{H}\left|\Psi\right\rangle = 0, \tag{1}$$

where $\widehat{H}$ is the Hamiltonian of the Universe given in an arbitrary frame of reference.

This theory uses Dirac's covariant approach to quantization of dynamical systems with constraints [3], with the physical state of the system determined by the conditions that the quantum constraints equal zero. As a result, the quantum state of the Universe does not depend on any parameters of the frame of reference, including the coordinate time (the problem of time in the 'frozen' formalism of quantum gravity theory [4, 5]). In this case, the time parameter should be sought among the dynamic variables of the theory. For this purpose, it seems natural to take the variables directly related to the expansion of the Universe, as they are contained in its scale factor

$$\Omega(x) = [\det g_{ik}(x)]^{1/6},$$

where $g_{ik}(x), i,k = 1,2,3$ is the field of the metric tensor of 3D geometry of the spatial section Σ.

This choice of cosmic time is the main one in analyzing the solutions of the Einstein equations near the cosmological singularity (where $\Omega(x) \to 0$) [6]. Below, we are going to give a general justification for this natural choice within the framework of the Hamiltonian formalism. Time is closely related to energy, so it should be expected that the solution of the time problem is also connected with the definition of energy in a closed Universe.

Eq. (1) has the form of a stationary Schrödinger equation with zero eigenenergy. In this paper, the zero energy of the Universe will be interpreted to mean that the positive energy of matter filling the Universe (this includes the energy of gravitational waves) is completely compensated by the negative energy of the gravitational field, associated with the scale factor, which arises in case of a closed space (a 100 % mass defect). First of all, the gravitational field energy in a closed Universe is divided into two parts with opposite signs: the part of the energy that includes the energy of gravitational waves in an expanding Universe, and the energy of space expansion itself. The proposed approach is based on the theorem of positive energy of the gravitational field, which was originally formulated for an island mass distribution with asymptotically flat space-time geometry [7, 8]. The Hamiltonian formulation of this result, together with the new complex canonical variables of general relativity (GR) theory, was proposed in [9]. Ref. [10] offered a generalization of this theorem to the case of a closed Universe, where the Hamiltonian had the form of a linear combination of GR constraints H_μ, $\mu = 0,1,2,3$ (the constraints are the densities of weight +1, see the Ref. [11] for their explicit form):

$$H = \int_\Sigma d^3 x N^\mu H_\mu. \quad (2)$$

The coefficients N^μ are called the lapse and shift functions, and determine the geometry of the (3+1)-partition of space-time into space and time. The Witten identity [7, 8] using auxiliary bispinor fields on the spatial section Σ takes the central place in this analysis.

In case of asymptotically flat space-time geometry, the auxiliary spinor field that is the solution of the Dirac equation on the spatial section enters directly into the expression for the energy of the gravitational field. It does not have the physical meaning of a force field but determines the equally important result of the action of the gravitational field that is the speed of time at each point of the spatial section [8]. In the case of a closed Universe, the Witten identity produces the sought representation of Hamiltonian (2) as a difference of two positive definite quantities:

$$H = h - D^2, \quad (3)$$

one of which, h, coincides in form with the positive energy of the gravitational field in asymptotically flat space-time, and the second, D^2, is connected with the scale factor $\Omega(x)$ and can be called the energy of the expansion of space or simply the energy of space. Representation (3) arises by using a special parametrization of the lapse and shift functions N^μ with the components of the Dirac bispinor field ψ on the spatial cross section Σ [12]. Now the components of the bispinor ψ determine the frame of reference and the corresponding geometry of space-time foliation, so we will call it the gauge spinor. Both terms in expression (3) are quadratic forms on the space of bispinor fields, in particular,

$$h = (\psi, \hat{h}\psi), \quad (4)$$

where the round brackets denote the scalar product in the space of bispinor fields.

In the presence of matter fields, their energy, proportional to the energy-momentum tensor, also enters quadratic form (4) with a positive sign [7]. From now on we will refer to this value as the energy of matter. According to the constraint equations,

$$H_\mu \approx 0, \quad (5)$$

Hamiltonian (2) is equal to zero, which means that the difference in the energies of matter and space in a closed Universe is equal to zero.

Having at our disposal the quantity of energy of matter in a closed Universe, with the value of this energy bounded from below, we can set the task of finding its minimum in quantum cosmology. This problem was formulated in [13], where it is proposed to regard the corresponding ground state as the Beginning of the Universe. In this paper, we have formulated the principle of the minimum (extremum) of the energy of matter and the equations that follow from it as a basis for the definition of physical states of the Universe in quantum cosmology, alternative to the WdW equation (1). In ordinary quantum mechanics, the stationary Schrödinger equation

$$\hat{h}|\Psi\rangle = E|\Psi\rangle \quad (6)$$

arises naturally in the problem of the minimum (extremum) of the mean energy value (see, for example, Ref. [14]):

$$\langle W \rangle = \frac{\langle \Psi | \hat{h} | \Psi \rangle}{\langle \Psi | \Psi \rangle} \quad (7)$$

on a Hilbert space of states of an isolated system.

This problem itself makes sense insofar as the energy operator $\widehat{h}$ is bounded from below. The closed Universe is an ideal isolated system, and according to the statement we have made above, the energy of matter in it is positive definite. Under quantization, quadratic form (4) becomes an operator

$$\widehat{h} = (\psi, \widehat{\widehat{h}}\psi), \tag{8}$$

acting in the space of states of the Universe $|\Psi\rangle$, where the two 'hats' now indicate the operator acting in the space of bispinors ψ and simultaneously in the space of states of the Universe.

Thus, the minimum (extremum) problem must be formulated for the functional

$$\rho = \frac{\langle\Psi|(\psi, \widehat{\widehat{h}}\psi)|\Psi\rangle}{\langle\Psi|(\psi,\psi)|\Psi\rangle}. \tag{9}$$

This quantity has the dimension of the energy density because the denominator in the normalizing quadratic form (ψ,ψ) involves integration over the volume of the Universe. However, the difficulty in formulating this minimum principle is that physical degrees of freedom of matter (and of gravitational waves) also interact with the scale factor $\Omega(x)$. The scale factor determines the volume of the Universe, but we have excluded it from the physical degrees of freedom. The interaction in question is that both terms on the right-hand side of expression (3) depend on all parameters of 3D geometry.

In order to take this dependence into account, it is sufficient to supplement the principle of the minimum of the mean energy density by additional conditions imposed by quantum constraints on the physical states of the Universe:

$$\langle\Psi|\widehat{H}_\mu|\Psi\rangle = 0. \ (10) \tag{10}$$

Notice that Gaussian constraints of gauge theories that underlie the standard model of matter [15] are added to gravitational constraints (5). Let us denote these Gaussian constraints as

$$G_a \approx 0, \ (11) \tag{11}$$

where a is the numbering index.

This addition occurs automatically, since constraints (11) are contained in the energy-momentum tensor of matter fields. As a result, we obtain the conditional principle of the minimum (extremum) of the mean energy of matter in a closed Universe for the functional

$$\frac{\langle\Psi|(\psi, \widehat{\widehat{h}}\psi)|\Psi\rangle}{\langle\Psi|(\psi,\psi)|\Psi\rangle} + \\ + \int_\Sigma \sqrt{g}\, d^3x [l^\mu \langle\Psi|\widehat{H}_\mu|\Psi\rangle + A^a \langle\Psi|D_a|\Psi\rangle], \tag{12}$$

where $l_\mu(x), A^a(x)$ are the Lagrange multipliers.

The following section contains the necessary definitions and the explicit form of representation (3) of the Hamiltonian of a closed Universe. An operator representation of gravitational constraints is derived from the Witten identity; the choice of the scale factor of the Universe as an internal ('multi-arrow') time parameter in the expanding Universe is substantiated. Then we obtain a system of equations, including the sought-for 'stationary' Schrödinger equation, from conditional minimum principle (12).

Hamiltonian of a closed Universe

Let us introduce the spinor variables present in the Witten identity and in the proof of the theorem on the positive energy of the gravitational field [7, 8]. A spinor field is a pair of complex numbers $\lambda_A \in C^2, A = 0,1,$ given at each point of the spatial section Σ, which has certain transformational properties under the transformation of spatial coordinates (the necessary definitions and notations of spinor algebra are given in [15]). When dealing with spinor variables, it is also convenient to use complex canonical variables (σ_{iAB}, M^{kCD}) for describing the gravitational field. These variables are defined by the relations

$$\sigma_{iAB}\sigma_k^{AB} = g_{ik},\ \sigma_{iAB}\sigma^{iCD} = \varepsilon_{(A}{}^C\varepsilon_{B)}{}^D, \tag{13}$$

$$M^{kCD} \equiv \pi^{kl}\sigma_l{}^{CD}, \tag{14}$$

and also by the Hermitian condition for the spin coefficients of the metric:

$$\sigma^+_{iAB} \equiv 2n_A{}^{C'}n_B{}^{D'}\bar{\sigma}_{iC'D'} = -\sigma_{iAB}, \tag{15}$$

where $\pi^{lm}(x),\ l,m = 1,2,3$ are the canonical impulses conjugate to the $3D$ metric tensor (they are the tensor density of weight +1); together with $g_{ik}(x)$ they form the real canonical variables of the gravitational Arnowitt – Deser – Misner field (ADM) (see [11]). The overbar indicates the operation of complex conjugation.

The spin tensors and $n^{AB'}$, determined by the relations $\varepsilon_{AB} = -\varepsilon_{BA}$, $\varepsilon_{01} = 1$ and

$$n^{AA'} n^{B}{}_{A'} = \frac{1}{2}\varepsilon^{AB}, \tag{16}$$

are, respectively, a metric tensor and an arbitrary unitary spin tensor in the space of spinors C^2. The spinor indices become superscripts or subscripts using ε_{AB} and acquire a prime through the operation of complex conjugation of the spinor (two primes annihilate).

Of all the canonical Poisson brackets (PB) for the new variables, let us concentrate on only one PB relation associated with the scale factor Ω of the Universe:

$$\{\ln\Omega^2(x), M(y)\} = \delta^3(x-y), \tag{17}$$

where $M \equiv M^{kCD}\sigma_{kCD}$.

The scale factor determines the 3D volume of the Universe:

$$V = \int_\Sigma \Omega^3 d^3x. \tag{18}$$

Let us introduce complex connection into the space of spinor fields [17]:

$$A_{kMN} = \Gamma_{kMN}(\sigma) + \frac{i}{\sqrt{2g}} M_{kMN}, \tag{19}$$

where $\Gamma_{lMN}(\sigma)$ is the ordinary torsion-free spin connection which obeys the Hermitian condition of the form (15).

It determines the covariant derivative

$$\nabla_k \lambda_A \equiv \partial_k \lambda_A + A_{kA}{}^{B}\lambda_B \tag{20}$$

in the space of spinor fields. We also require Dirac bispinors

$$\psi \in C^2 \otimes \bar{C}^2.$$

Let us define a 3D Dirac operator on the space of bispinor fields

$$\widehat{D}\psi \equiv i\sqrt{2}\begin{pmatrix} n_A{}^{B'}\sigma^k{}_{B'}{}^{C}\nabla_k\lambda_C \\ n_{A'}{}^{B}\bar{\sigma}^k{}_{B}{}^{C'}\bar{\nabla}_k\bar{\mu}_{C'} \end{pmatrix}, \quad \psi = \begin{pmatrix} \lambda_A \\ \bar{\mu}_{A'} \end{pmatrix} \tag{21}$$

and introduce a Hermitian scalar product

$$(\psi_1, \psi_2) \equiv \int_\Sigma \sqrt{g} d^3x n_{AA'} (\bar{\lambda}_1{}^{A'}\lambda_2{}^{A} + \mu_1{}^{A}\bar{\mu}_2{}^{A'}), \tag{22}$$

Dirac operator (21) is Hermitian with respect to this scalar product [12].

The Witten identity links the bilinear form of the squared Dirac operator $\widehat{D}^2$ with the bilinear form of another positive definite operator, which (with a slight modification) is the required representation of the energy of matter in a closed Universe.

Using the notations we have adopted and omitting for simplicity the contribution of matter fields, we can write this identity in the form [12]:

$$(\psi_1, \widehat{D}^2\psi_2) - (\psi_1, \widehat{w}\psi_2) = H[\psi_1, \psi_2], \tag{23}$$

where

$$(\psi_1, \widehat{w}\psi_2) \equiv \frac{1}{4}\int_\Sigma \sqrt{g} d^3x g^{ik} n_{AA'} \times \\ \times (\bar{\nabla}_i \bar{\lambda}_1{}^{A'} \nabla_k \lambda_2{}^{A} + \nabla_i \mu_1{}^{A} \bar{\nabla}_k \bar{\mu}_2{}^{A'}), \tag{24}$$

and the bilinear form on the right-hand side is a linear combination of gravitational constraints.

In case of an asymptotically flat space-time geometry, the Witten identity makes it possible to express the energy of the island system, which is determined by the surface integral at spatial infinity through a positive definite quadratic form which is obtained from expression (24) with $\psi_1 = \psi_2 = \psi$, where the bispinor ψ is a solution of the Dirac equation

$$\widehat{D}\psi = 0 \tag{25}$$

with the given asymptotic value of ψ_0.

We should note that Witten actually did not expect an auxiliary spinor variable obeying differential equation (25) to appear in general relativity, in addition to the already existing fields of matter [7]. In this case, the bispinor ψ, as previously indicated, specifies the local properties of the globally inertial frame of reference in which the energy is determined.

In case of a closed Universe, there is no surface integral and no usual definition of energy, and the physical consequences of identity (23) will be somewhat different. Let us now take an arbitrary bispinor field $\psi \neq 0$ on the space section Σ, and, assuming that $\psi_1 = \psi_2 = \psi$ in identity (23), write the representation of the Hamiltonian of a closed Universe (in a frame of reference governed by ψ) as a difference of two positive definite quadratic forms:

$$H[\psi] = (\psi, \widehat{w}\psi) - (\psi, \widehat{D}^2\psi). \qquad (26)$$

Here the Hamiltonian $H[\psi]$ is given as a gauge [11]:

$$N^0 = \frac{1}{4\sqrt{2}} n_{AA'}(\lambda^A \overline{\lambda}^{A'} + \mu^A \overline{\mu}^{A'}), \qquad (27)$$

$$N^k = -\frac{1}{4}\sigma^k{}_{AB} n^B{}_{A'}(\lambda^A \overline{\lambda}^{A'} + \mu^A \overline{\mu}^{A'}). \qquad (28)$$

Thus representation (3) is proved. Being quadratic relative to the canonical momenta of matter and the geometry of space-time, the Hamiltonian of a closed Universe also produces the geometry of the configuration space of GR (superspace). It is evident from the representation of Hamiltonian (26) that this geometry is pseudo-Riemannian in any frame of reference.

Let us now propose a physical interpretation of both terms on the right-hand side of (26). The first term is nonzero for any ψ, since Dirac operator (21) is an elliptic operator on a compact manifold Σ, so that its spectrum is discrete and separated from zero. Notice that this operator contains only the momentum $M \equiv M^{kCD}\sigma_{kCD}$, which, according to relation (17), is canonically conjugate to the scale factor of 3D geometry (more precisely, to the value $\ln(\Omega^2)$. For this reason, we shall refer to this contribution to the Hamiltonian as the energy of expansion of the 3D space of the Universe. However, the momentum M, which is proportional to Ω and determines the kinetic energy of the expansion of space, is also contained in the second quadratic form of the Hamiltonian (26). Its contribution can be 'extracted' from there in a covariant manner by introducing completely symmetric spin tensors [12]

$$\varphi^{MNA} \equiv \sigma^{iMN}\nabla_i\lambda^A + \frac{2}{3}\varepsilon^{A\{M}\sigma^{iN)}{}_P\nabla_i\lambda^P, \qquad (29)$$

$$\chi^{MNA} \equiv \sigma^{iMN}\nabla_i\mu^A + \frac{2}{3}\varepsilon^{A\{M}\sigma^{iN)}{}_P\nabla_i\mu^P. \qquad (30)$$

It is easy to verify that these spin tensors do not contain the canonical momentum M. Now the representation of the Hamiltonian of a closed Universe takes the following final form:

$$h[\psi] - \frac{11}{9}(\psi, \widehat{D}^2\psi) = H[\psi], \qquad (31)$$

where

$$h[\psi] \equiv (\psi, \widehat{h}\psi) = \frac{1}{2}\int_\Sigma \sqrt{g}\, d^3x\, n_{AA'} n_{MM'} n_{NN'} \times \\ \times (\overline{\varphi}^{A'M'N'}\varphi^{AMN} + \overline{\chi}^{A'M'N'}\chi^{AMN}) + (\psi, \widehat{T}\psi). \qquad (32)$$

Here the 'hats' denote the operators acting in the space of bispinor fields.

We have added the contribution of ordinary matter proportional to the corresponding energy-momentum tensor [7] to this expression for the energy of the gravitational field and will treat all of this together as the energy of the physical degrees of freedom in a closed Universe. The first term in (32) represents the part of the energy of the gravitational field that includes the energy of gravitational waves in the expanding Universe.

We should note that this expression for the energy in a closed Universe coincides with that obtained for the island mass distribution with the asymptotically flat geometry of space-time, since in this case the bispinor ψ obeys the Dirac equation (25). This can be seen from the operator form of (32):

$$\widehat{h} = -\frac{1}{2}\Delta + \frac{2}{9}\widehat{D}^2 + \widehat{T}, \qquad (33)$$

where

$$\Delta\psi \equiv \frac{1}{\sqrt{g}}\begin{pmatrix} n_A{}^{A'}\overline{\nabla}_i(\sqrt{g}g^{ik}n_{A'}{}^B\nabla_k\lambda_B) \\ n_{A'}{}^A\nabla_i(\sqrt{g}g^{ik}n_A{}^{B'}\overline{\nabla}_k\overline{\mu}_{B'}) \end{pmatrix} \qquad (34)$$

is the Beltrami–Laplace operator in the space of bispinor fields.

The first term on the right-hand side of equality (33) is, like this entire expression, a positive definite operator and represents the operator of the energy of the gravitational field in case of asymptotically flat space-time geometry.

Witten's identity (23) itself, which is valid for any ψ_1, ψ_2, leads to an operator equality on the space of bispinor fields, namely,

$$\widehat{h} - \frac{11}{9}\widehat{D}^2 = 0, \qquad (35)$$

if gravitational constraints (5) are satisfied.

Equality (35) can be solved explicitly with respect to the Dirac operator:

$$\widehat{D} = \pm\sqrt{\frac{9}{11}\widehat{h}}, \qquad (36)$$

where the square root of a positive definite Hermitian operator (33) can also be defined as a Hermitian operator.

Here the operators act in the space of bispinor fields. Since the Dirac operator is linear with respect to M, we can conclude that the system of gravitational constraints is solved with respect to the canonical momenta conjugate to the dynamic variable $\ln(\Omega^2(x)$.

This representation of gravitational constraints can be used as a basis for a noncovariant form of quantum gravity with classical 'multi-arrow' time [10] whose role is naturally played by the $\ln(\Omega^2(x)$ variable (there is an arrow of time for each point x of the spatial section Σ).

However, operator equation (36) does not in itself describe the evolution of the Universe until a specific observer is fixed. In quantum theory this equation only takes the form of a nonstationary Schrödinger equation with a single cosmic time parameter in the projection onto an arbitrary gauge spinor $\psi(x)$:

$$(\psi, \widehat{D}\psi) = \pm\left(\psi, \sqrt{\frac{9}{11}\widehat{h}}\psi\right). \tag{37}$$

This is a clear violation of covariance, since the gauge spinor is fixed arbitrarily. In the next section, we offer an alternative description of quantum dynamics of the Universe without an explicit violation of covariance. Instead of extracting the square root in operator equation (35) and introducing the classical time parameter, associated with the scale factor $\Omega(x)$ and 'projected' onto an arbitrary gauge spinor $\psi(x)$, we formulate a quantum theory based on the principle of minimum (extremum) energy of matter (12), in which the scale factor is quantized along with the other components of the 3D metric.

Quantization of energy density in a closed Universe

Let us carry out the quantization in a standard manner, by replacing the canonical momenta p by the operators of variational differentiation on the space of states of the Universe $|\Psi\rangle$:

$$p(x) \equiv \frac{\hbar}{i}\frac{\delta}{\delta q(x)}, \tag{38}$$

where $q(x) \equiv (\sigma_{iAB}(x), \phi(x))$ $(\phi(x))$ is the set of matter fields.

Respectively, the operators in this space of states are the Hamiltonian of matter $\widehat{h}$, determined by substituting operators (38) into Hamiltonian (31) and expression (32), and gravitational constraints $\widehat{H}_\mu$. We should note that Gaussian constraints (11) with the corresponding Lagrange multipliers are already contained in the second term of operator (32). At this stage, there is a difficulty in determining these operators due to the ambiguity of the ordering of the non-commuting operator multipliers. We shall here regard this difficulty as technical and shall not discuss it in detail.

After the basic quantities are determined, the remaining operation is to write down the equations that follow from the conditional extremum principle (12). The variational parameters are the wave function of the Universe $\Psi[\sigma, \phi]$, the gauge bispinor $\psi(x)$, and the Lagrange multipliers $l_\mu(x), A^a(x)$.

The variation with respect to $|\Psi\rangle$ gives the actual Schrödinger equation:

$$\frac{(\psi, \hat{\hat{h}}\psi)}{(\psi,\psi)}|\Psi\rangle + \int_\Sigma \sqrt{g}d^3x[\tilde{l}^\mu \widehat{H}_\mu|\Psi\rangle + \widetilde{A}^a D_a|\Psi\rangle] = \rho|\Psi\rangle, \tag{39}$$

where the mean value of the double energy operator $\hat{\hat{h}}$ is calculated in the frame of reference determined by the gauge spinor $\psi(x)$. It remains an operator in the space of states $|\Psi\rangle$ of the Universe.

The modified Lagrange multipliers $\tilde{l}_\mu(x), \widetilde{A}^a(x)$ differ from the initial ones by a normalization factor

$$\frac{\langle\Psi|(\psi,\psi)|\Psi\rangle}{(\psi,\psi)}. \tag{40}$$

Variation with respect to $\psi(x)$ gives an equation for the gauge spinor:

$$\frac{\langle\Psi|\hat{\hat{h}}\,\Omega^3(x)|\Psi\rangle}{\langle\Psi|\Omega^3(x)|\Psi\rangle}\psi = \rho\psi, \tag{41}$$

where the mean value of $\hat{\hat{h}}$ is calculated in the space of states of the Universe with the weight $\Omega^3(x)$ and is an operator in the space of bispinor fields on Σ.

Thus, the distribution of energy of matter, determined by the operator $\hat{\hat{h}}$, fixes the frame of reference corresponding to a given state of the Universe. Finally, the variation with respect to Lagrange multipliers l_μ, A_0^a gives constraint equations (10) plus additional gauge constraints

$$\langle\Psi|G_a|\Psi\rangle = 0. \quad (42)$$

The constraint equations are sufficient for determining the Lagrange multipliers in equality (35); after that the self-consistent system of equations (39) and (41) forms a ‘stationary’ Schrцdinger equation in quantum cosmology.

We should note that for a fixed $|\Psi\rangle$, the operator acting on $\psi(x)$ on the left-hand side of equality (41), is, as well as $\widehat{D}^2$, elliptic on a compact manifold Σ. Its spectrum is discrete. We can assume that the spectrum of eigenvalues of ρ in this problem is also discrete, since $|\Psi\rangle$ and $\psi(x)$ are found self-consistently.

Thus, the parameter ρ that enters into equations (39) and (41) is numbered by some set of quantum numbers. In classical cosmology, the energy density of matter in an expanding Universe decreases with the passage of cosmic time, starting from an infinite value. In the quantum theory considered here, the solutions of the ‘stationary’ Schrцdinger equation differ by the degree of excitation of the physical degrees of freedom of matter. There is a state of minimal excitation among them, which is called the basic state of the Universe in quantum cosmology in [13], where it was proposed as the Beginning of the Universe. In this state, the energy density of matter is maximal but finite.

Now it becomes necessary to interpret the entire set of solutions of the ‘stationary’ Schrцdinger equation. The basic assumption that we make about the spectrum of admissible values of ρ is that they are realized in the quantum evolution of the Universe. This means that the cosmic time should be sought in the above-mentioned set of quantum numbers. In this picture of quantum evolution of the Universe, the time parameter is discrete. The continuous nature of evolution with continuous time should be expected, as usual, at high degrees of excitation of the energy of matter. Another element of the formalism is the additional field of the calibration spinor $\psi(x)$ which determines the local properties of the frame of reference that accompanies the distribution of matter at a given moment in cosmic time. The lapse function $N^0(x)$ constructed from it, according to gauge (27), sets the speed of the standard clock located in each point of space. Thus, together with cosmic time, a frame of reference to which this time should be assigned is also fixed. There is no violation of covariance, since the parameters of the frame of reference are determined self-consistently with the actual distribution of matter in this quantum state. However, consistent development of this interpretation assumes knowledge of the structure of the quantum energy density spectrum.

Conclusion

In this paper, we have proposed to consider the quantum evolution of the Universe in terms of the energy parameter, the mean energy density of matter. The basis for the new approach is that the dynamic structure of the general theory of relativity in the case of a closed Universe allows defining the quantities of the energy of matter (including gravitational waves) and the energy of space expansion. In any frame of reference given by the gauge spinor ψ, these quantities are sign-definite quadratic forms of ψ that cancel each other out if gravitational constraints are satisfied.

Thus, the concept of a closed Universe as an object with a 100 % mass-energy defect has been substantiated. In this case, the quantization of the theory can be based on the principle of minimum energy with respect to one part of the internal energy of the Universe, namely, to the energy of matter and gravitational waves.

As a result, a system of equations is obtained which serves as an equivalent of the stationary Schrцdinger equation in ordinary quantum mechanics for a closed Universe. The system includes Eq. (39), whose solutions are the states of the Universe $|\Psi\rangle$ with a certain mean energy density ρ. It is supplemented by Eq. (41) for the gauge spinor ψ, i.e., the frame of reference to which the physical state $|\Psi\rangle$ should be assigned, and the quantization parameter is the value of the mean energy density ρ. The quantum-constraint equations (10) and (42) fix the uncertain Lagrange multipliers in Schrцdinger’s equation (39). In this formalism there is no violation of covariance, since additional calibration conditions are not necessary for fixing the Lagrange multipliers. In contrast to the conventional covariant form of quantum theory based on the Wheeler – de Witt equations (1), observable parameters ρ and $\psi(x)$ are present here, which together can be linked to cosmic time and the frame of reference in quantum cosmology. The cosmic time is discrete in this formalism. Continuous evolution of the Universe should be expected only in the late stages corresponding to a high degree of excitation of the energy of matter.

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