ELECROMAGNETIC WAVE PROPAGATION IN THE THREE-LAYER FERRITE-DIELECTRIC-FERRITE STRUCTURE

A.S. Cherepanov

Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russian Federation

A problem of wave propagation in a rectangular waveguide containing a three-layer ferrite-dielectric-ferrite (FDF) structure has been considered in the paper. The calculation of a free-space FDF structure usually runs into difficulties. The proposed approach has been made it possible to obtain a rigorous solution for the waveguide modes for which there is no dependence of electromagnetic fields on a coordinate directed along a magnetizing magnetic field. It is the main mode of the FDF structure that governs principal properties of a phased array. The obtained relationships were shown to describe the modes' behavior for a free-space FDF waveguide. The dependences of the mode propagation constants on the magnetizing magnetic field were calculated, electromagnetic field structures of the main and the higher modes were found. The optimal structure parameters were determined. They are optimal when the controllability of the FDF structure by a constant magnetic field is maximal.

Key words: ferrite; waveguide; antenna; dielectric structure; magnetizing magnetic field

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Introduction

To date, fabricating phased antenna arrays with optimal technical parameters and low cost remains an important problem. One possible solution is using electrically controlled ferrite-dielectric-ferrite (FDF) structures to construct integrated phased array antennas (IPAAs) [1 – 3]. Such antenna arrays have a simple design, so they can be manufactured by integrated technology methods, which considerably reduces the production costs.

An FDF structure is an open waveguide operating with multimode operation. Electrodynamic analysis of such a structure is complicated because the waveguide is open (unshielded) and contains magnetized ferrite which is a non-reciprocal medium. For these reasons, the analysis can be performed only approximately.

Finding a strict dispersion equation describing the properties of at least the basic type of waves in such a waveguide is an interesting task (it is the properties of the main mode of waves in the FDF that determine the most important characteristics of the IPAA). This allows to gain a better understanding of the physical properties of the proposed structure and to optimize the antenna.

In this paper, we have considered a three-layer FDF structure in a closed rectangular waveguide. It is known that the distribution of electromagnetic waves in such waveguides can be described rigorously in a number of cases [4, 5]. Below we are going to prove that for a high dielectric constant of a dielectric plate in an FDF structure, the electromagnetic field outside this structure decreases quite rapidly, so that the presence of the walls near the waveguide has virtually no effect on the main mode.

Problem statement and solution

Let us consider the problem of a rectangular waveguide with two ferrite plates and a dielectric plate located between them (Fig. 1). The external bias magnetic field is directed along the z axis (the ferrite plates are magnetized in the opposite direction).

The magnetic permeability tensor of ferrite is written in the form

$$\hat{\mu} = \begin{bmatrix} \mu & i\mu_a & 0 \\ -i\mu_a & \mu & 0 \\ 0 & 0 & \mu_\parallel \end{bmatrix},$$

where i is the imaginary unit, μ, μ_a, $\mu_\parallel$ are the components of the magnetic permeability tensor [4, 7, 8].

It was established in [4] that if the fields do not depend on the z coordinate, only the electric field components E_z and the magnetic field components H_x- and H_y- remain nonzero in a rectangular waveguide. The component E_z in the ferrite satisfies the equation

$$\frac{d^2 E_z}{dx^2} + \nu_f^2 E_z = 0, \quad (1)$$

where $\nu_f^2 = k^2 \varepsilon_f \mu_\perp - \beta^2$ (k is the wavenumber of the empty space, ε_f is the dielectric permittivity of the ferrite, β is the sought-for constant of wave propagation in the waveguide;

$$\mu_\perp = \frac{\mu^2 - \mu_a^2}{\mu}.$$

The components of the magnetic field in ferrite can be found using the following relations [4]:

$$H_x = \frac{1}{k\mu_\perp}\left(\beta E_z - \frac{\mu_a}{\mu}\frac{dE_z}{dx}\right), \quad (2)$$

$$H_y = \frac{j}{k\mu_\perp}\left(\beta \frac{\mu_a}{\mu} E_z - \frac{dE_z}{dx}\right). \quad (3)$$

Relations (1) – (3) describe the fields of regions II and IV (ferrite plates). Similar relations are also satisfied for region III (dielectric), and for regions I and V (air).

The only difference is the change in material parameters. For example, instead of the quantity ν_f^2, the quantity

$$\nu_d^2 = k^2 \varepsilon_d - \beta^2$$

can be written for the dielectric, where ε_d is the dielectric permittivity of region III.

In the air-filled regions I and V, $\nu_0^2 = k^2 - \beta^2$ instead of ν_f^2. Similarly, the form of relations (2) and (3) changes in these regions.

We can write the solution of Eq. (1) for all five regions and apply the boundary conditions to them: that the component E_z be equal to zero on the side walls and that the tangential components of the fields (E_z and H_y) be continuous at the interface between the regions.

In view of the above, we can write the fields for the waveguide regions I – V (see Fig. 1):

$$\text{I.}\ E_z = A\sin(\nu_0 x),$$

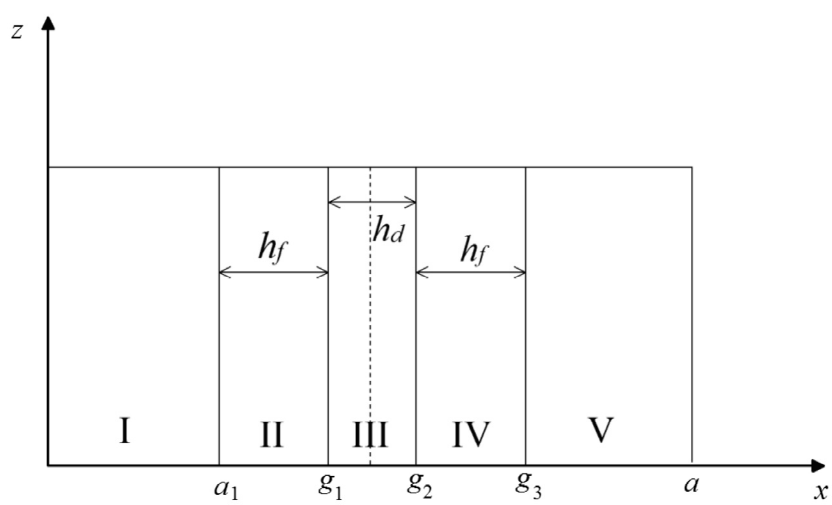

Fig. 1. Schematic representation of the FDF structure in a rectangular waveguide: a is the waveguide width; a_1, g_1 – g_3 are the other geometric parameters; h_f, h_d are the widths of ferrite (II, IV) and dielectric (III) plates, respectively; regions I, V are filled with air; the external magnetic field is directed along the z axis

$$\text{II.}\ E_z = B\sin[\nu_f(x-a_1)] + C\cos[\nu_f(x-a_1)],$$

$$\text{III.}\ \begin{aligned} E_z &= D\sin[\nu_d(x-a_1-h_f)] + \\ &+ E\cos[\nu_d(x-a_1-h_f)], \end{aligned}$$

$$\text{IV.}\ \begin{aligned} E_z &= F\sin[\nu_f(x-a_1-h_f-h_d)] + \\ &+ G\cos[\nu_f(x-a_1-h_f-h_d)], \end{aligned}$$

$$\text{V.}\ E_z = I\sin[\nu_0(a-x)].$$

The boundary conditions on the side walls have already been taken into account here.

As a result, we obtain a homogeneous system of linear equations of the eighth order with unknown coefficients *A*, *B*, *C*, *D*, *E*, *F*, *G*, *I*. A non-trivial solution of this system exists only if its determinant is equal to zero:

$$\det\hat{Z} = \\ = \det\begin{bmatrix} z_{AA} & 0 & z_{AC} & 0 & 0 & 0 & 0 & 0 \\ z_{BA} & z_{BB} & z_{BC} & 0 & 0 & 0 & 0 & 0 \\ 0 & z_{CB} & z_{CC} & 0 & z_{CE} & 0 & 0 & 0 \\ 0 & z_{DB} & z_{DC} & z_{DD} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & z_{ED} & z_{EE} & 0 & z_{EG} & 0 \\ 0 & 0 & 0 & z_{FD} & z_{FE} & z_{FF} & z_{FG} & 0 \\ 0 & 0 & 0 & 0 & 0 & z_{GF} & z_{GG} & z_{GI} \\ 0 & 0 & 0 & 0 & 0 & z_{IF} & z_{IG} & z_{II} \end{bmatrix} = \\ = 0. \quad (4)$$

Thus, we have obtained a dispersion equation that allows to find the propagation constants for the modes of the three-layer waveguide, first and foremost, for the main mode. The resulting equation is rigorous, that is, the accuracy of the solution is determined only by the accuracy of the procedure for calculating its roots.

The nonzero elements of the matrix $\hat{Z}$ are given below:

$$z_{AA} = \sin(\nu_0 a_1),\ \ z_{AC} = -1;$$

$$z_{BA} = -\nu_0\cos(\nu_0 a_1),\ \ z_{BB} = \nu_f / \mu_\perp,$$

$$z_{BC} = -\beta\mu_a / \mu\mu_\perp;$$

$$z_{CB} = \sin(\nu_f h_f),\ \ z_{CC} = \cos(\nu_f h_f),\ \ z_{CE} = -1;$$

$$z_{DB} = \frac{1}{\mu_\perp}\left[\frac{\beta\mu_a}{\mu}\sin(\nu_f h_f) - \nu_f\cos(\nu_f h_f)\right],$$

$$z_{DC} = \frac{1}{\mu_\perp}\left[\frac{\beta\mu_a}{\mu}\cos(\nu_f h_f) - \nu_f\cos(\nu_f h_f)\right],$$

$$z_{DD} = \nu_d;$$

$$z_{ED} = \sin(\nu_d h_d),\ \ z_{EE} = \cos(\nu_d h_d).\ \ z_{EG} = -1;$$

$$z_{FD} = \nu_d\cos(\nu_d h_d),\ \ z_{FE} = -\nu_d\sin(\nu_d h_d),$$

$$z_{FF} = -\nu_f / \mu_\perp.\ \ z_{FG} = \beta\mu_a / \mu\mu_\perp.$$

$$z_{GF} = \sin(\nu_f h_f).\ \ z_{GG} = \cos(\nu_f h_f),$$

$$z_{GI} = -\sin(\nu_0 a_1);$$

$$z_{IF} = \frac{1}{\mu_\perp}\left[-\frac{\beta\mu_a}{\mu}\sin(\nu_f h_f) - \nu_f\cos(\nu_f h_f)\right],$$

$$z_{IG} = \frac{1}{\mu_\perp}\left[-\frac{\beta\mu_a}{\mu}\cos(\nu_f h_f) + \nu_f\cos(\nu_f h_f)\right],$$

$$z_{II} = -\nu_0\cos(\nu_0 a_1).$$

Investigation of the waveguide modes of the FDF structure

By solving dispersion equation (4), we can obtain the dependence of the propagation constant β on the off-diagonal term of the tensor $\hat{\mu}$ (i.e., in fact, on the bias magnetic field), as well as on other parameters. Then, if we use the data of [4 – 6, 2, 3], we can find the structure of the fields for each mode. A combination of parameters of the FDF structure making it possible to create a workable IPAA was proposed and experimentally tested in [6]. However, no conclusions could be drawn as to whether this combination of parameters was optimal. It is possible to carry out such an investigation now that we have obtained an analytical solution for the main mode of an FDF waveguide.

FDF waveguides whose parameters are given in Table 1 were considered in Ref. [6].

Fig. 2 shows the dependence of the deceleration $q = \beta / k$ for two modes of the FDF waveguide, obtained using the parameters from Table 1, on the magnitude of the off-diagonal term μ_a of the tensor $\hat{\mu}$ (the remaining elements of the tensor are assumed to be equal to unity). The result corresponds to that obtained in [6].

Fig. 3 shows the dependence of the controllability of the FDF waveguide

$$\Delta q = (\beta_{\max} - \beta_{\min}) / k$$

Table 1

Parameters of the FDF waveguides used in [6]

ka	$h_f\ /\ a$	$h_d\ /\ a$	ε_f	ε_d
4.82	0.097	0.042	12	40

Notations: h_f, h_d are the widths of the ferrite and dielectric plates, respectively; ε_f, ε_d are their dielectric permeabilities; k is the wavenumber of the empty space; a is the width of the rectangular waveguide.

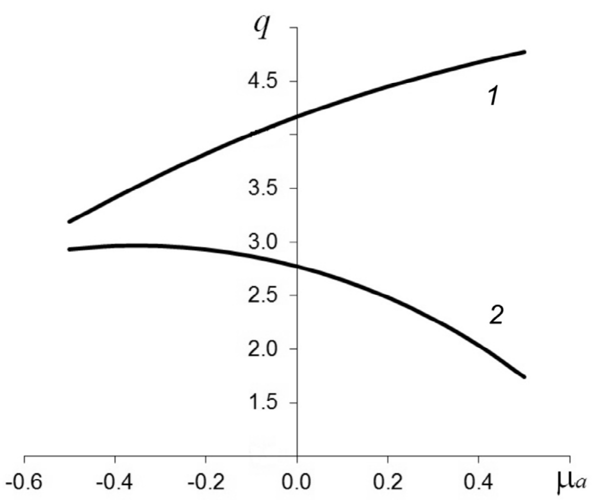

Fig. 2. Dependence of the deceleration $q = \beta / k$ of the main (1) and second (2) modes of the FDF waveguide on the magnitude of the off-diagonal term μ_a of the tensor $\hat{\mu}$;; parameters from Table 1 were used

on the thickness $h_f\ /\ a$ of ferrite plates. The remaining parameters were assumed to be the same as before (see Table 1).

It can be seen from Fig. 3 that controllability Δq increases with increasing plate thickness but the growth rate drops sharply for thickness values $h_f\ /\ a > 0.1$. This is because ferrite effectively interacts with the electromagnetic wave only in the regions where the polarization of the magnetic field is close to circular, while the field's magnitude must be large enough. These regions are located near the dielectric plate. When thick ferrite plates are used, regions with a small magnetic field are involved. Therefore, a large increase in controllability is not achieved. A large amount of ferrite in the FDF waveguide leads to an increase in the waveguide's losses, as well as an increase in its weight. Therefore, it is expedient to choose the plate thickness by the inequality $h_f\ /\ a < 0.1.$. The value given in [6] seems valid.

Let us now investigate the dependence of controllability Δq on the thickness h_d of the dielectric. Fig. 4 shows the dependence of Δq on h_d / a (the other parameters are taken from Table 1). It can be seen from the graph that there is an optimal thickness value ensuring maximum controllability for the dielectric.

The physical meaning of the presence of a dielectric plate in an FDF waveguide is that it 'absorbs' the electromagnetic field so that the energy propagates along the waveguide inside the plate and in close proximity to it, i.e., where the ferrite is located. This is what provides good controllability. If the dielectric is too thick, the field is concentrated in it, and the ferrite has a small magnitude of the electromagnetic field, resulting in decreased controllability. If the dielectric is too thin, it cannot concentrate the field, a lot of energy propagates outside the ferrite, and controllability decreases as well.

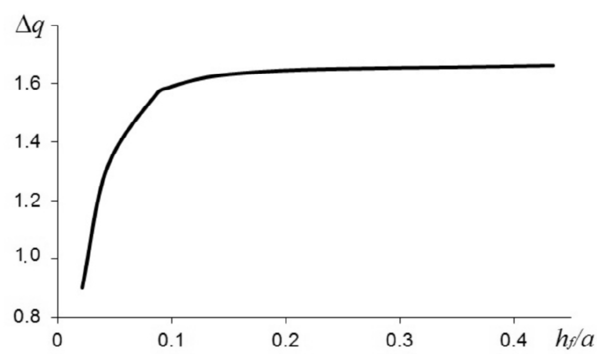

Fig. 3. Dependence of the controllability $\Delta q = (\beta_{max} - \beta_{min}) / k$ of the FDF waveguide on the normalized thickness $h_f\ /\ a$ of ferrite plates; parameters from Table 1 were used

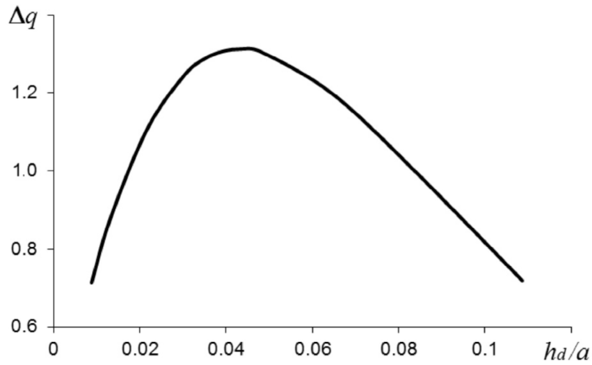

Fig. 4. Dependence of controllability Δq on the normalized thickness h_d/a of the dielectric; parameters from Table 1 were used

Fig. 5 shows the dependences of the normalized component of the electric field E_z on the coordinate x/a for the main and the second mode. It can be seen that the electromagnetic wave of the main mode is concentrated mainly in the FDF structure and decreases exponentially outside it. Therefore, the presence of side walls should not significantly affect the propagation constant of this mode. This is confirmed by direct calculation (the results are given in Table 2).

It can be seen from the data in Table 2 that deceleration remains constant to three decimal places with a change in the waveguide width a. It follows then that narrow walls of the waveguide can be 'continued to infinity', that is, removed completely. Wide walls can also be removed, since the electric field on them does not have a tangential component and the boundary conditions are not violated. We obtain an open waveguide where we can accurately calculate the propagation constant of the main mode. The structure of the field is known in the region between the wide walls. Outside these wide walls, the field can be reconstructed, since tangential components of the magnetic field on these walls are known.

Thus, the problem of calculating the main mode for an open FDF structure has actually been solved.

The second mode decreases much slower outside the FDF structure, so a larger waveguide width a should be taken to calculate it accurately.

Fig. 6 shows the graphs of the structure of magnetic fields for the demagnetized state ($\mu_a = 0$) and for the maximum values of magnetization ($\mu_a = \pm 0.5$). The fields are normalized by $H_{y\max}$ with $\mu_a = 0$. It can be seen that magnetic fields in ferrite are larger if $\mu_a = -0.5$ than if $\mu_a = 0.5$. In addition, the polarization of the magnetic field in ferrite plates is close to circular for $\mu_a = -0.5$, while for $\mu_a = 0.5$, the field in ferrite, especially in

Table 2

Set of parameters used and the result of the calculation of deceleration of electromagnetic waves propagating in the FDF structure

Parameter	Notation	Unit	Value
Width			
ferrite	h_f	cm	0.224
dielectric	h_d		0.096
Off-diagonal term of the tensor of magnetic permeability of ferrite [7, 8]	μ_a	–	+ 0.5
Dielectric constant			
ferrite	ε_f	–	12
dielectric	ε_d		40
Microwave frequency	f	GHz	10
Waveguide width	a	cm	2.3; 3.0; 4.0; 5.0
The obtained value of deceleration $q = \beta / k$ is 4.774 for all given parameter values			

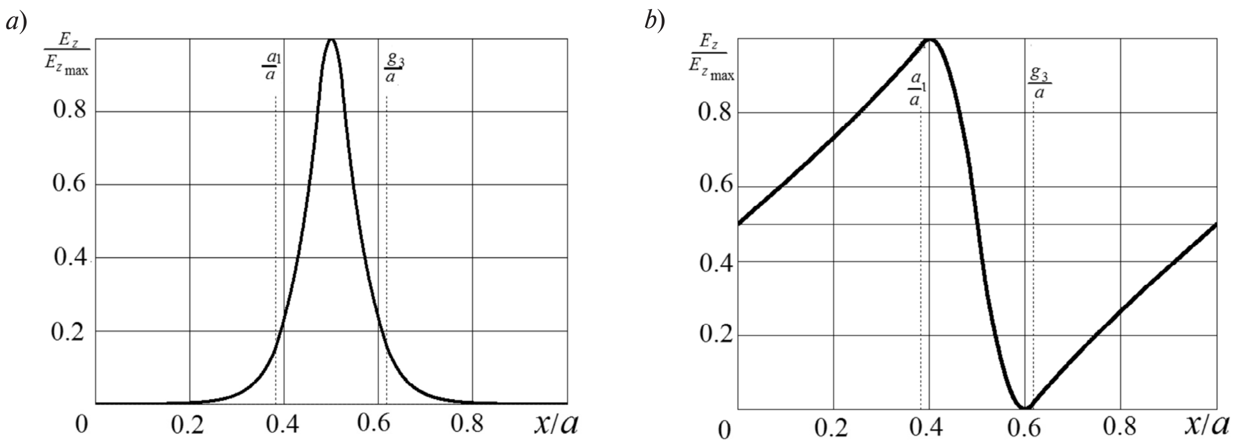

Fig. 5. Dependences of the normalized component of the electric field E_z on the normalized coordinate x for the main (*a*) and the second (*b*) mode.
The geometric parameters correspond to those shown in Fig. 1

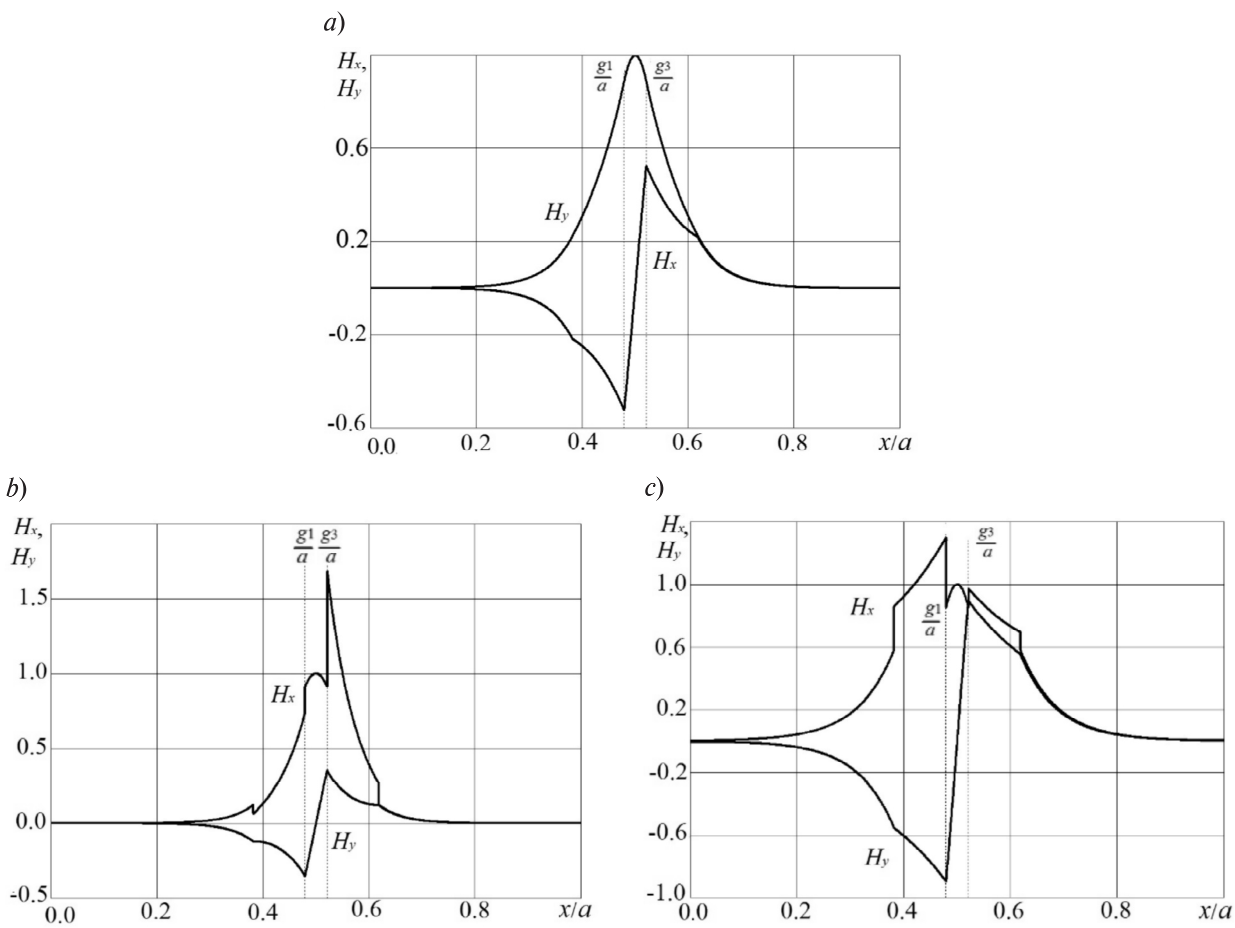

Fig. 6. Results of calculation of the magnetic field distributions (the H_x and H_y components) in the FDF structure for its demagnetized state (*a*) and for the maximum values of magnetization (*b*, *c*); the magnetic permeability $\mu_a = 0$ (*a*); + 0.5 (*b*); −0.5 (*c*).
The fields are normalized by $H_{y\max}$ with $\mu_a = 0$

the right-hand plate, differs greatly from circularly polarized. Therefore, as can be seen from the graph in Fig. 2, the slope of the $q(\mu_a)$ dependence is steeper for negative μ_a.

Conclusion

The problem of propagation of waves in a rectangular waveguide containing a three-layer ferrite-dielectric-ferrite structure has been solved rigorously in the study. The proposed approach allows to investigate the modes of electromagnetic oscillations with no dependence of the fields on the coordinate directed along the bias field. We have established that the obtained relations also describe oscillation modes for an open FDF waveguide. In an open FDF structure, it is the main oscillation mode that determines the key properties of the integrated phased array antennas, such as the scanning angle and the beamwidth. Applying the proposed approach to investigation of an open FDF waveguide eliminates the need for cumbersome numerical methods taking considerable computational time. This greatly simplifies calculating the performance of integral phased antenna arrays.

The results obtained can also be used to create ferrite phase shifters [9, 10] based on a closed rectangular waveguide with a three-layer FDF structure.

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THE AUTHOR

CHEREPANOV Andrey S.

Peter the Great St. Petersburg Polytechnic University

29 Politechnicheskaya St., St. Petersburg, 195251, Russian Federation

ASCherSPb@mail.ru